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SHORTER COMMUNICATION

SURFACE RADIATION PROPERTIES FROM ELECTROMAGNETIC THEORY*

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NOMENCLATURE

k,	absorption index;
n,	refractive index;
N,	complex index of refraction.

Greek symbols

α, β, γ,	parameters defined in equation (2);
ϵ,	emissivity;
θ,	polar angle;
ρ,	reflectivity.

Subscripts

$\perp$,	perpendicular–polarized component;
$\parallel$,	parallel–polarized component;
N,	evaluated in direction of surface normal;
H,	hemispherical.

ELECTROMAGNETIC theory provides expressions for the specular reflectance of optically smooth surfaces which are physically and chemically uncontaminated in terms of the optical indices of the material, polarization of the incident radiation, and the angle of incidence. The angular dependence of the exact relationships is sufficiently complex to render radiative transfer studies including polarization and directional effects tedious. Furthermore, the exact expressions generally require numerical integration to evaluate the associated hemispherical emissivity. The purposes of this study are threefold. First, to compare hemispherical emissivity values determined by numerical integration of the exact relationships and by evaluation of an approximate closed form analytical expression. Second, to construct figures for the ratio of hemispherical emissivity to normal emissivity of sufficient detail to be useful for the commonly observed values for the optical parameters of engineering materials. Finally, to establish criteria the use of which assures certain degrees of accuracy in simplified formulae for the directional reflectivity components, directional reflectivity, and hemispherical emissivity.

The expressions from electromagnetic theory [1] for the specular reflectivity of a non-magnetic material for incident radiation polarized perpendicular to the plane of incidence, $\rho_\perp(\theta)$, and polarized parallel to the plane of incidence, $\rho_\parallel(\theta)$, may be put in the following form:

$$\left.\begin{aligned}
\rho_\perp(\theta) &= \frac{(n\beta - \cos\theta)^2 + n^2[(1+k^2)\alpha - \beta^2]}{(n\beta + \cos\theta)^2 + n^2[(1+k^2)\alpha - \beta^2]} \\
\rho_\parallel(\theta) &= \frac{\left(n\gamma - \dfrac{\alpha}{\cos\theta}\right)^2 + n^2[(1+k^2)\alpha - \gamma^2]}{\left(n\gamma + \dfrac{\alpha}{\cos\theta}\right)^2 + n^2[(1+k^2)\alpha - \gamma^2]}
\end{aligned}\right\} \quad (1)$$

where

$$\left.\begin{aligned}
\alpha^2 &= \left[1 + \left(\frac{\sin^2\theta}{n^2(1+k^2)}\right)\right]^2 - \frac{4}{(1+k^2)}\left[\frac{\sin^2\theta}{n^2(1+k^2)}\right] \\
\beta^2 &= \frac{(1+k^2)}{2}\left[\left(\frac{1-k^2}{1+k^2}\right) - \left(\frac{\sin^2\theta}{n^2(1+k^2)}\right) + \alpha\right] \\
\gamma &= \frac{(1-k^2)}{(1+k^2)}\beta + \frac{2k}{(1+k^2)}[(1+k^2)\alpha - \beta^2]^{\frac{1}{2}}
\end{aligned}\right\} \quad (2)$$

and the angle θ denotes the polar angle of incidence relative to the surface normal. The reflectivity relations may be

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employed to calculate the polarized components of directional emissivity $\epsilon_\perp(\theta)[= 1 - \rho_\perp(\theta)]$ and $\epsilon_{\parallel}(\theta)[= 1 - \rho_{\parallel}(\theta)]$ provided the emitting material is of sufficient thickness to render it opaque. The refractive index n and the absorption index k are here defined in terms of the complex refractive index N as $N = n(1 - ik)$. Although not explicitly indicated, the above reflectivity relations yield spectral values since the optical parameters n and k are generally wave length dependent. Also, these expressions are strictly valid when the adjacent medium is vacuum or a gas with refractive index unity and absorption index zero.

When the incident radiation is uniformly polarized, the specular reflectivity, $\rho(\theta)$, is the arithmetic mean of the polarized components and the directional emissivity, $\epsilon(\theta)$, is given by $[1 - \rho(\theta)]$. The normal emissivity $\epsilon_N[= \epsilon(0)]$ follows from equation (1) as

$$\epsilon_N = \frac{4n}{(n+1)^2 + n^2k^2} \tag{3}$$

while the hemispherical emissivity, ϵ_H, is obtained by integration over half space

$$\epsilon_H = \int_0^1 \epsilon(\theta)\,\mathrm{d}(\sin^2\theta). \tag{4}$$

The indicated integration must be performed numerically for arbitrary values of the optical parameters. An exception occurs for perfect dielectrics ($k = 0$), in which case the following result obtains [2].

$$\epsilon_H = \frac{1}{2} - \frac{(3n+1)(n-1)}{6(n+1)^2} - \frac{n^2(n^2-1)^2}{(n^2+1)^3}\ln\left(\frac{n-1}{n+1}\right) + \frac{2n^3(n^2+2n-1)}{(n^2+1)(n^4-1)} - \frac{8n^4(n^4+1)}{(n^2+1)(n^4-1)^2}\ln(n). \tag{5}$$

Of the above, the reflectivity results have received the greatest attention and Holl [3] has recently presented a compilation of available tables and graphs for these.

It is evident from equations (1) and (2) that the complex angular dependence of the reflectivity expressions can be significantly reduced when $\sin^2\theta$ is small in comparison to the value for the parameter $n^2(1 + k^2)$. With such an approximation, α, β and γ are approximately unity and the reflectivity relationships reduce to the following:

$$\left.\begin{aligned} \rho_\perp(\theta) &= \frac{(n-\cos\theta)^2 + n^2k^2}{(n+\cos\theta)^2 + n^2k^2} \\ \rho_{\parallel}(\theta) &= \frac{\left(n - \dfrac{1}{\cos\theta}\right)^2 + n^2k^2}{\left(n + \dfrac{1}{\cos\theta}\right)^2 + n^2k^2}. \end{aligned}\right\} \tag{6}$$

The above approximate formulae continue to yield the correct result for normal emissivity, equation (3), and when utilized in equation (4), give the following closed form expression for hemispherical emissivity [2].

$$\begin{aligned} \epsilon_H = {} & 4n + \frac{4}{n(1+k^2)} - 4n^2\ln\left[\frac{n^2(1+k^2)+2n+1}{n^2(1+k^2)}\right] \\ & - \frac{4}{n^2(1+k^2)^2}\ln\left[n^2(1+k^2)+2n+1\right] \\ & + \frac{4n^2(1-k^2)}{k}\tan^{-1}\left[\frac{k}{n(1+k^2)+1}\right] \\ & + \frac{4(1-k^2)}{n^2k(1+k^2)^2}\tan^{-1}\left[\frac{nk}{n+1}\right]. \end{aligned} \tag{7}$$

Minimum values of $n^2(1 + k^2)$ have been determined which assure prescribed accuracy in the use of the approximate relations, but first we turn to the hemispherical emissivity results.

Hemispherical emissivities computed by numerical integration of the exact expression for directional emissivity and calculated from equation (7) are illustrated in Figs. 1 and 2 with absorption index as a parameter for refractive index values greater than and less than unity, respectively. The solid curves represent the numerical integration results and the dashed curves those obtained from the approximate analytical expression. For refractive index values greater than unity, typical of metals in the i.r. spectral region and of dielectrics, the discrepancy between results is indistinguishable on Fig. 1 when the absorption index exceeds 0·5. The largest discrepancy is about 9 per cent at $n = 1$ and $k = 0$ and diminishes with increasing values for absorption index. For refractive index values less than unity such as exhibited by metals in the u.v. and often the visible portion of the spectrum, errors as large as an order of magnitude may be incurred by the use of the approximate expression. Fortunately, most metals with refractive indices less than unity have large values for absorption index so that equation (7) is generally accurate except when the refractive index is less than 0·1. Criteria have been developed for the use of equation (7) in evaluating hemispherical emissivity in terms of the parameter $n^2(1 + k^2)$. An accuracy of not less than 1, 2, 5 and 10 per cent is assured in the calculation of hemispherical emissivity with equation (7) provided the value for $n^2(1 + k^2)$ exceeds 40, 3·25, 1·75 and 1·25, respectively. According to reported values for the optical indices of common metals in the visible and infrared spectral regions [4], the approximate hemispherical emissivity relationship yields values which are accurate to within 2 per cent.

The ratio of hemispherical emissivity to normal emissivity is of particular importance since it is normal emissivity which is more often measured. Figure 3 displays this ratio as calculated from the numerical integration results for a range of

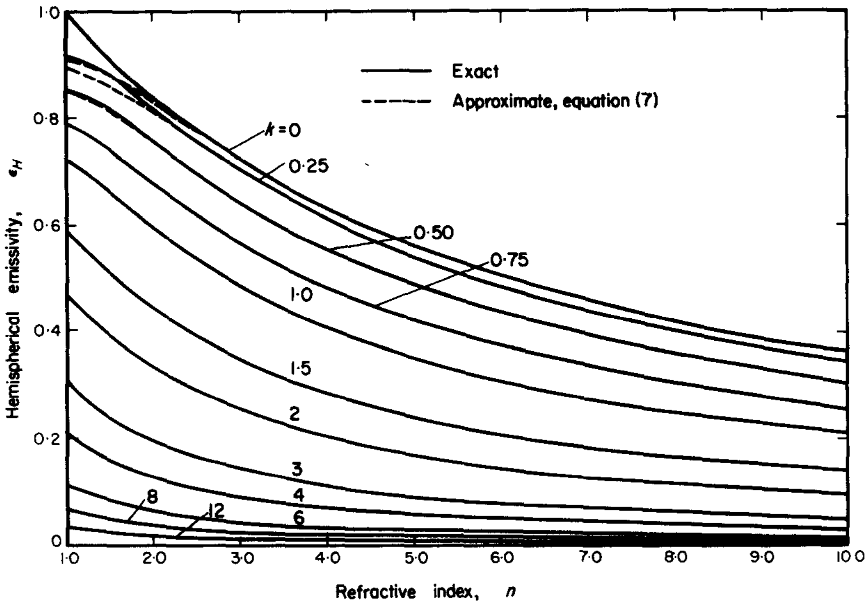

FIG. 1. Exact and approximate hemispherical emissivity results ($n \geqslant 1{\cdot}0$).

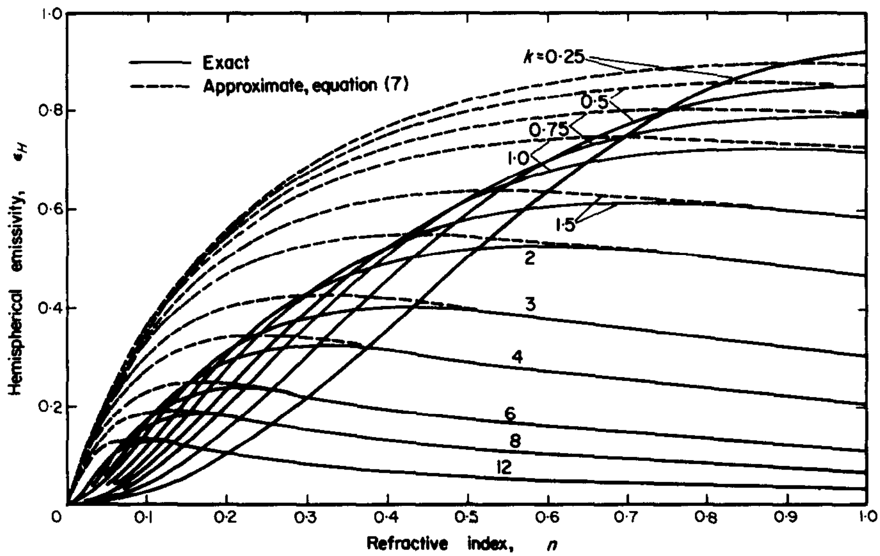

FIG. 2. Exact and approximate hemispherical emissivity results ($n \leqslant 1{\cdot}0$).

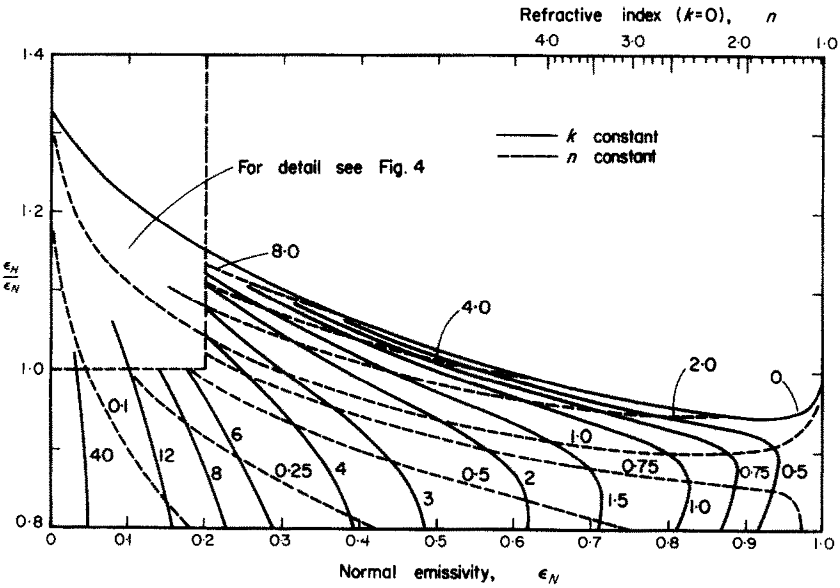

FIG. 3. Ratio of hemispherical emissivity to normal emissivity.

values for the optical parameters which includes those commonly observed [4] for metals in the u.v. and visible portions of the spectrum and for dielectrics. Excluding spectral regions in the neighborhood of absorption bands, the absorption index of dielectrics is essentially zero and therefore a measurement of monochromatic normal emissivity suffices to determine the monochromatic hemispherical emissivity as well as the refractive index. With the evaluation of the latter in mind, a refractive index scale has been included on Fig. 3 for the $k = 0$ curve.

In the infrared spectral region, metals generally have normal emissivity values less than 0·2 and values greater than unity for the emissivity ratio. Figure 4, also constructed from the integration results, illustrates the ϵ_H/ϵ_N ratio for this important range. Without prior knowledge of the values for the optical parameters, only an estimate of hemispherical emissivity of a metal can be made with an experimentally determined normal emissivity value. It is evident from Fig. 4 that the error in the estimate for hemispherical emissivity could exceed 30 per cent. Thus, accurate values for hemispherical emissivity of metals as well as for dielectrics near absorption bands can only be assured by supplementing normal emissivity data with data for the refractive and the absorption index at the wave length of the emissivity measurement. Conversely, should experimental hemispherical and normal emissivity values be available, Fig. 4 may be employed to estimate the optical parameters and hence the directional distribution of the emitted energy.

The accuracy of the approximate expressions for the specular reflectivity components as well as the specular reflectivity for uniformly polarized incident radiation was also examined. Minimum values for the parameter $n^2(1 + k^2)$ were established which assure certain selected accuracies in the evaluation of $\rho_\perp(\theta)$, $\rho_\parallel(\theta)$ and $\rho(\theta)$ for all angles of incidence. These are presented in Table 1. The approximate relations generally yield considerably greater accuracy for $\rho_\perp(\theta)$ and $\rho(\theta)$ than for $\rho_\parallel(\theta)$. Also, the value for $n^2(1 + k^2)$ required to assure selected accuracy in $\rho_\parallel(\theta)$ is strongly dependent on the minimum value for the absorption index considered in the development of the criteria. As a result, $n^2(1 + k^2)$ values determined for two selected minimum absorption index values are given. For common metals in the infrared spectral region, the approximate expressions yield directional reflectivity values within 1 per cent of those evaluated from the exact relations.

Table 1. Minimum values of $n^2(1 + k^2)$ for selected accuracy in approximate property expressions

Per cent Accuracy	1	5	10
ϵ_H	40	1·75	1·25
$\rho(\theta)$	20	7	5
$\rho_\perp(\theta)$	20	8	5
$\rho_\parallel(\theta)$*	370	200	100
$\rho_\parallel(\theta)$†	225	70	35

* Minimum k = 0·05.
† Minimum k = 0·25.

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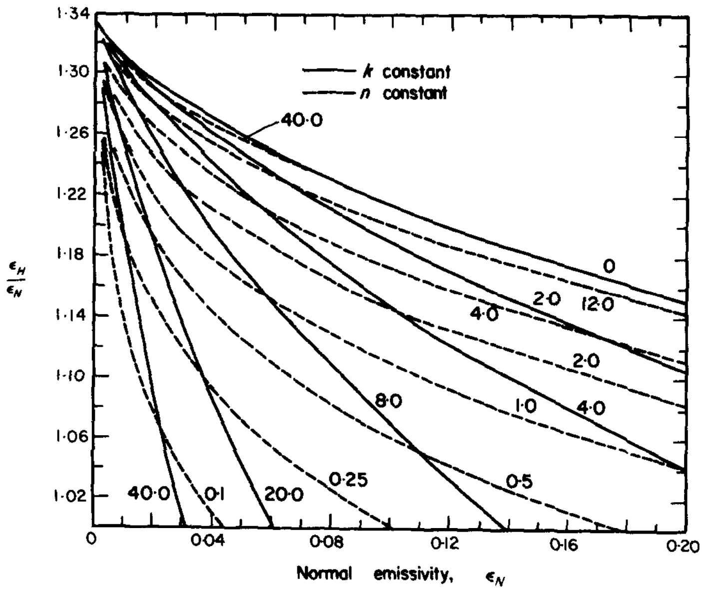

FIG. 4. Ratio of hemispherical emissivity to normal emissivity.